

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Algebra

Changing the subjects of formulae with two or more steps

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

Alternative method 1

$$3c = d + 2$$

$$\text{or } 3c - 2$$

M1

$$d = 3c - 2 \text{ or } d = -2 + 3c$$

$$\text{or } 3c - 2 = d \text{ or } -2 + 3c = d$$

A1

Alternative method 2

$$c - \frac{2}{3} = \frac{d}{3}$$

$$\text{or } 3\left(c - \frac{2}{3}\right)$$

M1

$$d = 3\left(c - \frac{2}{3}\right)$$

A1

Additional Guidance

Flow chart method, with incorrect final answer:

$$d \rightarrow +2 \rightarrow \div 3 \rightarrow c \text{ and } c \rightarrow \times 3 \rightarrow -2 \rightarrow d$$

M1A0

Condone $\times$ signs for M1 but not A1

Condone c3 for M1 but not A1

[2]

Q2.

Alternative method 1 – add 6 to both sides first

$$x + 6 = 2y \text{ or } -x - 6 = -2y \text{ or}$$

$$\frac{x+6}{2} \text{ or } \frac{x}{2} + 3 \text{ or } \frac{1}{2}(x+6) \quad \text{oe}$$

M1

$$y = \frac{x+6}{2} \text{ or } y = \frac{x}{2} + 3 \text{ or } y = \frac{1}{2}(x+6)$$

allow order reversed

do not allow further incorrect work e.g. attempts to divide only the 6 by 2

condone $y = (x+6) \div 2$ for M1A1

A1

Alternative method 2 – divide both sides by 2 first

$$\frac{x}{2} = y - \frac{6}{2} \text{ or } \frac{x}{2} = y - 3 \text{ or } \frac{x+6}{2} \text{ or } \frac{x}{2} + 3 \text{ or } \frac{1}{2}(x+6)$$

allow $\frac{2y}{2}$ for y

M1

$$y = \frac{1}{2}(x+6) \text{ or } y = \frac{x+6}{2} \text{ or } y = \frac{x}{2} + 3$$

allow order reversed

do not allow further incorrect work e.g. attempts to divide only the 6 by 2

condone $y = (x+6) \div 2$ for M1A1

A1

A1

Alternative method 3 – flow diagram

$$y \rightarrow 2y \rightarrow 2y - 6$$

$$\leftarrow x + 6 \leftarrow x$$

allow $2 \times y$ or $y \times 2$ for $2y$

ignore any operations seen on arrows

M1

$$y = \frac{x+6}{2} \text{ or } y = \frac{x}{2} + 3$$

$$\text{or } y = \frac{1}{2}(x+6)$$

allow order reversed

do not allow further incorrect work e.g. attempts to divide only the 6 by 2

condone $y = (x+6) \div 2$ for M1A1

A1

Additional Guidance

Allow 0.5 for $\frac{1}{2}$ throughout

[2]

Q3.

$$y\sqrt{x+1} = 1$$

or

$$\sqrt{x+1} = \frac{1}{y}$$

or

$$y^2 = \frac{1}{x+1}$$

M1

$$y^2(x+1) = 1$$

or

$$y^2x + y^2 = 1$$

or

$$y^2x = 1 - y^2$$

or

$$x + 1 = \frac{1}{y^2}$$

or

$$\frac{1}{y^2} - 1 \text{ or } \frac{1-y^2}{y^2}$$

M1dep

$$x = \frac{1}{y^2} - 1$$

or

$$x = \frac{1-y^2}{y^2}$$

or in the form $x =$

A1

Additional Guidance

Correct answer in working repeated on answer line without $x =$

eg $x = \frac{1}{y^2} - 1$ seen in working with answer $\frac{1}{y^2} - 1$

M1M1A1

Allow $\left(\frac{1}{y}\right)^2$ for $\frac{1}{y^2}$ throughout

Allow 1^2 for 1 throughout

[3]

Q4.

$$3h = g + 1 \text{ or } g + 1 = 3h$$

or

$$h - \frac{1}{3} = \frac{g}{3} \text{ or } \frac{g}{3} = h - \frac{1}{3}$$

or

$$\frac{g+1}{3} \text{ or } \frac{g}{3} + \frac{1}{3}$$

allow negative equivalents

eg $-3h = -g - 1$

correct rearrangement omitting $h =$

M1

$$h = \frac{g+1}{3} \text{ or } h = \frac{g}{3} + \frac{1}{3}$$

oe fully simplified

$$\text{SC1 } h = \frac{g-1}{3} \text{ or } h = \frac{g}{3} - \frac{1}{3} \text{ oe}$$

A1

Additional Guidance

$$\frac{g+1}{3} = h \text{ or } \frac{g}{3} + \frac{1}{3} = h$$

M1A1

Not fully simplified correct rearrangement eg $h = \frac{-g-1}{-3}$

M1A0

Correct solution followed by further incorrect simplification

M1A0

[2]

Q5.

Alternative method 1

$$v - u = at \quad -at = u - v$$

$$t = \frac{v-u}{a} \quad t = \frac{u-v}{-a} \quad \text{oe}$$

M1

A1

Alternative method 2

$$\frac{v}{a} = \frac{u}{a} + t$$

$$t = \frac{v}{a} - \frac{u}{a} \quad \text{oe}$$

M1

A1

Additional Guidance

$$t = (v - u) \div a$$

M1A1

$$v - u = at \text{ and } t = v - u \div a$$

M1A0

$$\frac{v-u}{a} \text{ or } \frac{u-v}{-a} \text{ or } \frac{v}{a} - \frac{u}{a}$$

M1A0

$$a = \frac{v-u}{t} \text{ with or without working}$$

M1A0

$$t = v - u \div a$$

M0A0

$$t = \frac{v+u}{a}$$

M0A0

[2]

Q6.

$$y - 9 = \frac{x}{3} \text{ or } 3y = x + 27 \text{ or } 3y - 27 \text{ or } 3(y - 9)$$

correct first step in rearranging
or the correct rearrangement without $x =$

M1

$$x = 3y - 27 \text{ or } x = 3(y - 9)$$

$$\text{Accept } 3y - 27 = x$$

$$\text{or } 3(y - 9) = x$$

A1

Additional Guidance

Accept $-27 + 3y$ for $3y - 27$ throughout

$x = 3y - 27$ in working with answer $3y - 27$

M1A1

$$x = (y - 9)3 \text{ (unless recovers)}$$

M1A0

$$x = y3 - 27 \text{ (unless recovers)}$$

M1A0

Multiplication signs are acceptable for M1 but not A1

$$x = 3 \times y - 27$$

M1A0

$$3 \times y = x + 3 \times 9$$

M1

[2]

Q7.

$$w = \frac{y}{2x}$$

B1

[1]

Q8.

$$y^2 = \frac{x}{2} + 1$$

M1

$$y^2 - 1 = \frac{x}{2} \text{ or } 1 - y^2 = -\frac{x}{2}$$

or

$$2y^2 = x + 2$$

or

$$2y^2 - 2 \text{ or } 2(y^2 - 1)$$

M1dep

$$x = 2y^2 - 2 \text{ or } x = 2(y^2 - 1)$$

A1

Additional Guidance

$x = 2y^2 - 2$ in working with $2y^2 - 2$ on answer line

M1M1A1

[3]

Q9.

Alternative method 1

$$ac = b + 5c$$

oe fraction eliminated

M1

$$ac - 5c = b \text{ or } c(a - 5) = b$$

$$\text{or } \frac{b}{a-5}$$

oe terms in c collected

M1dep

$$c = \frac{b}{a-5}$$

A1

Alternative method 2

$$a - 5 = \frac{b}{c}$$

M1

$$\frac{1}{a-5} = \frac{c}{b} \text{ or } \frac{a-5}{b} = \frac{1}{c}$$

$$\text{or } c(a - 5) = b$$

$$\text{or } \frac{b}{a-5}$$

M1dep

$$c = \frac{b}{a-5}$$

A1

Additional Guidance

$c = \frac{b}{a-5}$ in working lines with $\frac{b}{a-5}$ on answer line

M1M1A1

[3]

Q10.

$$x = \frac{y+2}{3}$$

B1

[1]

Q11.

$$w = \sqrt[3]{y^2}$$

B1

[1]

Q12.

(a) $\frac{165 + 567}{12}$ or $\frac{732}{12}$ oe

M1

61

SC1 212.25

A1

Additional Guidance

Only $165 + 567 \div 12$ with brackets missing

M0A0

61.00

M1A1

61.0

M1A0

(b) **Alternative method 1**

$50 = \frac{165+x}{15}$ or 50×15 or 750 seen
oe eg $750 = 165 + \text{cost of minibus}$
any letter or symbol or word(s)

M1

$50 \times 15 - 165$ oe

M1dep

585

SC1 915

A1

Alternative method 2

$165 \div 15$ or 11 oe

M1

$(50 - \text{their 11}) \times 15$ or 39×15 oe

M1dep

585

SC1 915

A1

Additional Guidance

Up to M2 may be awarded for correct work with no answer, or incorrect answer, even if this is seen amongst multiple attempts

$(165 + \text{any value}) \div 15$ does not imply M1 unless set up as an equation for the first mark of Alt 1

Allow 12 as a misread for 15

[5]

Q13.

(a) $0.2(20\,000 - 10\,600)$
or 0.2×9400
oe

M1

1880

A1

(b) 10 600

B1**Additional Guidance**

Accept 10 600.01 or 10 600.02 or 10 600.03 or 10 600.04

(c) **Alternative method 1**

$5200 = 0.2(E - 10\,600)$
oe

M1

$5200 \div 0.2$

or 26 000

oe 5200×5

M1dep

36 600

A1**Alternative method 2**

$5200 = 0.2(E - 10\,600)$
oe

M1

$5200 + 0.2 \times 10600$

or $5200 + 2120$

or 7320

M1dep

36 600

A1**[6]**